

SYMMETRIES AND STAGGERING EFFECTS IN NUCLEAR ROTATIONAL SPECTRA

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Abstract

We study the fine structure of nuclear rotational spectra on the basis of both dynamical and discrete symmetry approaches. In this framework we show that the odd–even ($\Delta I = 1$) staggering effects observed in various rotational bands carry detailed information about the collective band-mixing interactions and the collective shape properties of heavy nuclei.

1 Introduction

The odd–even staggering effect is known as a zigzagging behavior of the nuclear inertial parameter between the odd and the even angular momentum states of a rotational band. It due to a relative displacement of the odd angular momentum levels with respect to the even ones [1]. The analysis and the interpretation of this effect are of current interest since it carries detailed information about the fine properties of nuclear collective dynamics in different regions of nuclear chart.

We study the odd–even staggering effects in nuclear spectra in terms of an appropriately defined quantity

$$\text{Stg}(I) = 6\Delta E(I) - 4\Delta E(I-1) - 4\Delta E(I+1) + \Delta E(I+2) + \Delta E(I-2), \quad (1)$$

which is the discrete approximation of the fourth derivative of the function $\Delta E(I) = E(I+1) - E(I)$, i.e. the fifth derivative of the rotational band energy $E(I)$. In the present paper it will be shown that the above quantity is very sensitive to the fine structure of rotational spectra and provides spectacular $\Delta I = 1$ staggering patterns (zigzagging behavior of the function $\text{Stg}(I)$ with clearly defined zero reference) in various rotational bands. On this basis

we propose relevant theoretical analysis of the $\Delta I = 1$ staggering effects in collective γ rotational bands and nuclear octupole bands in reference to the underlying symmetries of nuclear collective interactions.

2 SU(3) dynamical symmetry and $\Delta I = 1$ staggering in heavy deformed nuclei

We have found [2] that Eq. (1) provides well developed staggering patterns in the γ bands of the nuclei ^{156}Gd , $^{156,160,162}\text{Dy}$, $^{162-166}\text{Er}$, ^{170}Yb and $^{228,232}\text{Th}$. We demonstrated that the observed effect can be interpreted as the result of the interaction of the γ band with the ground band in the framework of a Vector Boson Model with SU(3) dynamical symmetry [3]. In this model the two bands are coupled into the same SU(3) multiplet, which provides the following γ - band energy expression [2]:

$$E^\gamma(I) = 2B + AI(I+1) + B[\sqrt{1 + aI(I+1) + bI^2(I+1)^2} - CI(I+1) - 1] \left(\frac{1 + (-1)^I}{2} \right), \quad (2)$$

where the quantities A , B , C , a and b are determined by the effective model interaction. Eq. (2) reproduces successfully the $\Delta I = 1$ effect in all considered nuclei. In Fig. 1(a) the experimental and the theoretical staggering patterns obtained for the γ band of ^{166}Er are illustrated.

Our theoretical analysis of the staggering patterns observed in rare earth nuclei provide detailed information about the fine behavior of the ground- γ band mixing interaction in dependence on the nuclear shell structure in rotational regions. In addition, it suggests a detailed test and relevant comparison of the different kinds of dynamical symmetry schemes.

3 Octahedron point symmetry and $\Delta I = 1$ staggering in octupole bands

We propose a study of the fine structure of collective rotational bands with a presence of octupole degrees of freedom through the formalism of the octahedron (O) point symmetry group. Based on the irreducible representations

(irreps) of this group [4], we have constructed a collective Hamiltonian of a system with octupole correlations: $\hat{H}_{oct} = \hat{H}_{A_2} + \sum_{r=1}^2 \sum_{i=1}^3 \hat{H}_{F_r(i)}$ with

$$\hat{H}_{A_2} = a_2 \frac{1}{4} [(\hat{I}_x \hat{I}_y + \hat{I}_y \hat{I}_x) \hat{I}_z + \hat{I}_z (\hat{I}_x \hat{I}_y + \hat{I}_y \hat{I}_x)] , \quad (3)$$

$$\hat{H}_{F_1(1)} = \frac{1}{2} f_{11} \hat{I}_z (5 \hat{I}_z^2 - 3 \hat{I}^2) , \quad (4)$$

$$\hat{H}_{F_1(2)} = \frac{1}{2} f_{12} (5 \hat{I}_x^3 - 3 \hat{I}_x \hat{I}^2) , \quad (5)$$

$$\hat{H}_{F_1(3)} = \frac{1}{2} f_{13} (5 \hat{I}_y^3 - 3 \hat{I}_y \hat{I}^2) , \quad (6)$$

$$\hat{H}_{F_2(1)} = f_{21} \frac{1}{2} [\hat{I}_z (\hat{I}_x^2 - \hat{I}_y^2) + (\hat{I}_x^2 - \hat{I}_y^2) \hat{I}_z] , \quad (7)$$

$$\hat{H}_{F_2(2)} = f_{22} (\hat{I}_x \hat{I}^2 - \hat{I}_x^3 - \hat{I}_x \hat{I}_z^2 - \hat{I}_z^2 \hat{I}_x) , \quad (8)$$

$$\hat{H}_{F_2(3)} = f_{23} (\hat{I}_y \hat{I}^2 + \hat{I}_z^2 \hat{I}_y + \hat{I}_y^3 - \hat{I}_y \hat{I}^2) . \quad (9)$$

Here, the first term \hat{H}_{A_2} belongs to the one-dimensional irrep A_2 of the octahedron group O , while the terms $\hat{H}_{F_1(i)}$ and $\hat{H}_{F_2(i)}$ ($i = 1, 2, 3$) belong to its three-dimensional irreps F_1 and F_2 respectively, with a_2 and f_{ri} ($r = 1, 2$; $i = 1, 2, 3$) being the Hamiltonian parameters.

After taking into account the simultaneous presence of quadrupole degrees of freedom as well as the high order quadrupole–octupole interaction we applied the above Hamiltonian to obtain the rotational spectrum of the system by minimizing subsequently its energy with respect to the third angular momentum projection K for each given value of the angular momentum I .

We found that the so obtained energy bands after being used in Eq. (1) exhibit various staggering patterns in dependence on the model parameters. Several schematic examples are demonstrated in Fig. 1(b)-(d). On this basis we suppose that the model can be applied to reproduce the staggering effects in nuclear octupole bands [5] as well as in some rotational negative parity bands built on octupole vibrations .

4 Conclusion

The approaches suggested give a rather general prescription for analysis of various fine characteristics of rotational motion in quantum mechanical sys-

tems. They allow a detailed comparison of the effects of the collective interactions and shapes in different regions of nuclei. We propose a systematic study of the symmetries associated with these effects and suggest that it could provide a relevant guide in the revealing of some general properties of collective motion in the less studied regions, such as the exotic nuclei.

Acknowledgments

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Figure captions

Fig. 1 $\Delta I = 1$ staggering patterns: (a) for the γ band of ^{166}Er (theory and experiment); (b)–(d) for the schematic spectra obtained by the octahedron Hamiltonian [Eqs (3)–(9), theory].